

Chapter 5 Surface Area of Cylinders and Composite Solids

Teacher Guide and Diagnostic Answer Key

Teacher Guide

Teacher Chapter-at-a-Glance

A cylinder label or paper net is helpful
Pacing
Talk Prompt

Mastery Decision Guide

A missed Tripwire does not automatically mean “redo the chapter.” Confirm the likely misconception, reteach the smallest prerequisite or representation that addresses it, and then use a fresh item before moving on.

Missed Tripwire #1 → Diameter is being substituted where the formula requires radius. Next move: Write $d=14$, then $r=7$ before any formula; circle the r in the formula.

Confirmation and Fresh Recheck

Confirm with: Ask the student to shade exposed surfaces and cross out hidden contact faces before calculating.

Fresh recheck: Use an open cylinder or two joined cubes with new dimensions; require exposed-surface reasoning.

Decision rule: Move forward only when the fresh response is secure and the reasoning survives the changed representation or context.

Diagram Production Notes — Chapter 5

Use the diagram master IDs:

Ch16-D01 — Cylinder Net

Ch16-D02 — Curved Surface as Rectangle

Ch16-D03 — Closed Cylinder Face Plan

Ch16-D04 — Diameter Given Cylinder

Ch16-D05 — Closed / Open-Top / Tube

Ch16-D06 — Hidden Contact Faces

Ch16-D07 — Composite Solid Method

Ch16-D08 — Cylinder Surface Area vs Volume Trap

The cylinder net must clearly show two circular bases and one rectangle. The rectangle length must be labelled as $2\pi r$ and the height as h . Hidden contact faces in composite solids should be shaded or marked so students can see why they are not counted. Avoid crowded labels around circles.

Ready to Learn Check

Quick check before starting. A wrong response below points to a prerequisite worth confirming before the chapter builds on it.

1. What does circumference measure?

Correct answer: Distance around the circle.

Likely wrong answer: Space inside.

Possible misconception: Circumference and area are being confused.

Next move: Trace the rim and name it distance around.

2. What is the area of a circle with radius 3 cm?

Correct answer: 28.26 cm^2

Likely wrong answer: 18.84 cm or 9.42 cm^2

Possible misconception: Circle area or squaring is not secure.

Next move: Use $A = \pi r^2$ with $r = 3$ and square before multiplying.

3. What does surface area measure?

Correct answer: Outside covering.

Likely wrong answer: Space inside.

Possible misconception: Surface area and volume are being confused.

Next move: Ask whether paint touches the outside or fills the inside.

4. If two cubes are glued face to face, are the touching faces on the outside?

Correct answer: No. The touching faces are hidden.

Likely wrong answer: Yes, count every face from both cubes.

Possible misconception: Exposed and hidden faces are not being distinguished.

Next move: Physically press two cubes together and identify the faces you can no longer touch.

Ready-to-Learn item 1

Correct answer: Distance around the circle.

Likely wrong answer: Space inside.

Misconception: Circumference and area are being confused.

Next move: Trace the rim and name it distance around.

Ready-to-Learn item 2

Correct answer: 28.26 cm^2

Likely wrong answer: 18.84 cm or 9.42 cm^2

Misconception: Circle area or squaring is not secure.

Next move: Use $A = \pi r^2$ with $r = 3$ and square before multiplying.

Ready-to-Learn item 3

Correct answer: Outside covering.

Likely wrong answer: Space inside.

Misconception: Surface area and volume are being confused.

Next move: Ask whether paint touches the outside or fills the inside.

Ready-to-Learn item 4

Correct answer: No. The touching faces are hidden.

Likely wrong answer: Yes, count every face from both cubes.

Misconception: Exposed and hidden faces are not being distinguished.

Next move: Physically press two cubes together and identify the faces you can no longer touch.

Practice

Assign the full set or select items for targeted review. Teacher solutions follow each question.

1. radius 2 cm, height 7 cm

Solution: 1. $SA = 2(3.14)(2^2) + 2(3.14)(2)(7)$ $SA = 25.12 + 87.92$ $SA = 113.04 \text{ cm}^2$

2. radius 4 m, height 10 m

Solution: 2. $SA = 2(3.14)(4^2) + 2(3.14)(4)(10)$ $SA = 100.48 + 251.2$ $SA = 351.68 \text{ m}^2$

3. radius 5 cm, height 8 cm

Solution: 3. $SA = 2(3.14)(5^2) + 2(3.14)(5)(8)$ $SA = 157 + 251.2$ $SA = 408.2 \text{ cm}^2$

4. diameter 12 cm, height 10 cm Find the surface area.

Solution: 4. $r = 12 \div 2 = 6$ $SA = 2(3.14)(6^2) + 2(3.14)(6)(10)$ $SA = 226.08 + 376.8$ $SA = 602.88 \text{ cm}^2$

5. open-top cylinder with radius 3 cm, height 8 cm

Solution: 5. Open-top cylinder: $SA = \pi r^2 + 2\pi rh$ $SA = 3.14(3^2) + 2(3.14)(3)(8)$ $SA = 28.26 + 150.72$ $SA = 178.98 \text{ cm}^2$

6. tube with radius 4 m, height 9 m

Solution: 6. Tube: $SA = 2\pi rh$ $SA = 2(3.14)(4)(9)$ $SA = 226.08 \text{ m}^2$

7. open-top cylinder with diameter 10 cm, height 6 cm

Solution: 7. $r = 10 \div 2 = 5$ $SA = \pi r^2 + 2\pi rh$ $SA = 3.14(5^2) + 2(3.14)(5)(6)$ $SA = 78.5 + 188.4$ $SA = 266.9 \text{ cm}^2$

8. Two cubes with side length 4 cm are glued face to face. Find the total surface area.

Solution: 8. One cube: $SA = 6(4^2) = 6(16) = 96$ Two cubes before subtracting: $96 + 96 = 192$ One contact face: $4 \times 4 = 16$ Two hidden faces: $2 \times 16 = 32$ Composite surface area: $192 - 32 = 160$ 160 cm^2

9. Two cubes with side length 5 m are glued face to face. Find the total surface area.

Solution: 9. One cube: $SA = 6(5^2) = 6(25) = 150$ Two cubes before subtracting: $150 + 150 = 300$ One contact face: $5 \times 5 = 25$ Two hidden faces: $2 \times 25 = 50$ Composite surface area: $300 - 50 = 250$ 250 m^2

10. A cube with side length 3 cm is glued to the top of a rectangular prism. The contact face is $3 \text{ cm} \times 3 \text{ cm}$. The rectangular prism has surface area 150 cm^2 . The cube has surface area 54 cm^2 . Find the composite surface area.

Solution: 10. $150 + 54 = 204$ Contact face: $3 \times 3 = 9$ Two hidden contact faces: $2 \times 9 = 18$ Composite surface area: $204 - 18 = 186$ 186 cm^2

11. A student uses $V = \pi r^2 h$ to find the surface area of a cylinder. Explain the mistake.

Solution: 11. $V = \pi r^2 h$ finds volume, or space inside. Surface area counts outside covering.

12. A student finds the surface area of two glued cubes by adding both full surface areas.

Explain what is missing.

Solution: 12. The touching faces are hidden, so they must be subtracted. The contact area must be subtracted twice.

13. A student says a tube has two circular bases. Explain why the question wording matters.

Solution: 13. A tube has no circular ends. A closed cylinder has two circular bases. The wording tells which surfaces exist.

14. A closed cylinder has diameter 14 cm. A student uses 14 cm as the radius. Explain the mistake.

Solution: 14. Diameter is the full distance across the circle. Radius is half the diameter, so the radius is 7 cm, not 14 cm.

Chapter Answer Key

Tripwire Check

The student is not correct.

The diameter is 14 cm, so the radius is:

$$r = 14 \div 2$$

$$r = 7$$

Use:

$$SA = 2\pi r^2 + 2\pi rh$$

$$SA = 2(3.14)(7^2) + 2(3.14)(7)(9)$$

Two bases:

$$2(3.14)(49) = 307.72$$

Curved surface:

$$2(3.14)(7)(9) = 395.64$$

Total:

$$307.72 + 395.64 = 703.36$$

The surface area is:

$$703.36 \text{ cm}^2$$

Worksheet 1: Guided Practice

1.

a. 2

b. 1

2.

Two bases:

$$2(3.14)(4) = 25.12$$

Curved surface:

$$2(3.14)(2)(5) = 62.8$$

Total:

$$25.12 + 62.8 = 87.92$$

Surface area:

$$87.92 \text{ cm}^2$$

3.

$$r = 4$$

4.

a. No

b. No

Worksheet 2: Independent Practice

1.

a.

$$SA = 2(3.14)(3^2) + 2(3.14)(3)(4)$$

$$SA = 56.52 + 75.36 = 131.88$$

$$131.88 \text{ cm}^2$$

b.

$$SA = 2(3.14)(5^2) + 2(3.14)(5)(7)$$

$$SA = 157 + 219.8 = 376.8$$

$$376.8 \text{ m}^2$$

c.

$$r = 8 \div 2 = 4$$

$$SA = 2(3.14)(4^2) + 2(3.14)(4)(6)$$

$$SA = 100.48 + 150.72 = 251.2$$

$$251.2 \text{ cm}^2$$

d.

$$r = 14 \div 2 = 7$$

$$SA = 2(3.14)(7^2) + 2(3.14)(7)(10)$$

$$SA = 307.72 + 439.6 = 747.32$$

$$747.32 \text{ m}^2$$

2.

a.

$$SA = 3.14(2^2) + 2(3.14)(2)(6)$$

$$SA = 12.56 + 75.36 = 87.92$$

$$87.92 \text{ cm}^2$$

b.

$$SA = 3.14(4^2) + 2(3.14)(4)(5)$$

$$SA = 50.24 + 125.6 = 175.84$$

$$175.84 \text{ m}^2$$

c.

$$r = 12 \div 2 = 6$$

$$SA = 3.14(6^2) + 2(3.14)(6)(8)$$

$$SA = 113.04 + 301.44 = 414.48$$

$$414.48 \text{ cm}^2$$

3.

a.

$$SA = 2(3.14)(3)(10)$$

$$SA = 188.4 \text{ cm}^2$$

b.

$$SA = 2(3.14)(6)(5)$$

$$SA = 188.4 \text{ m}^2$$

c.

$$r = 10 \div 2 = 5$$

$$SA = 2(3.14)(5)(12)$$

$$SA = 376.8 \text{ cm}^2$$

4.

a. One cube:

$$6(3^2) = 54$$

Two cubes:

$$54 + 54 = 108$$

Contact face:

$$3 \times 3 = 9$$

Subtract twice:

$$108 - 18 = 90$$

$$90 \text{ cm}^2$$

b. One cube:

$$6(6^2) = 216$$

Two cubes:

$$216 + 216 = 432$$

Contact face:

$$6 \times 6 = 36$$

Subtract twice:

$$432 - 72 = 360$$

$$360 \text{ m}^2$$

Worksheet 3: Mixed Practice & Application

1.

a. Surface area, because paint covers the outside.

b.

$$SA = 2(3.14)(4^2) + 2(3.14)(4)(12)$$

$$SA = 100.48 + 301.44$$

$$SA = 401.92$$

$$401.92 \text{ cm}^2$$

2.

a. The container has no top, so only the bottom circular base is counted.

b.

$$SA = 3.14(5^2) + 2(3.14)(5)(9)$$

$$SA = 78.5 + 282.6$$

$$SA = 361.1$$

$$361.1 \text{ cm}^2$$

3.

The tube has no circular ends, so use curved surface area only.

$$r = 6 \div 2 = 3$$

$$SA = 2\pi rh$$

$$SA = 2(3.14)(3)(20)$$

$$SA = 376.8$$

The cardboard area is:

$$376.8 \text{ cm}^2$$

4.

a.

$$SA = 6(4^2) = 96$$

One cube has surface area 96 cm^2 .

b.

$$96 + 96 = 192$$

c.

$$4 \times 4 = 16$$

d.

$$192 - 2(16) = 192 - 32 = 160$$

The final surface area is:

$$160 \text{ cm}^2$$

5.

The student used the diameter as the radius.

The radius is:

$$r = 10 \div 2 = 5$$

Now find the surface area:

$$SA = 2(3.14)(5^2) + 2(3.14)(5)(6)$$

$$SA = 157 + 188.4$$

$$SA = 345.4$$

The correct surface area is:

$$345.4 \text{ cm}^2$$

6.

a. Surface area formula for a closed cylinder:

$$SA = 2\pi r^2 + 2\pi rh$$

b. Volume formula:

$$V = \pi r^2 h$$

c. Surface area measures outside covering. Volume measures space inside. Paint needs surface area. Filling the cylinder needs volume.

Diagnostic Key — High-Leverage Items

Tripwire Check

Correct answer: No. The diameter is 12 cm, so $r=6$ cm. For a closed cylinder, $SA=2\pi r^2+2\pi rh \approx 602.88 \text{ cm}^2$.

Likely wrong answer: Using $r=12$ and producing an answer based on 12^2 .

Misconception: Diameter is being substituted where the formula requires radius.

Next move: Write $d=12$, then $r=6$ before any formula; circle the r in the formula.

Tripwire Diagnostic

Correct answer: No. The diameter is 14 cm, so $r=7$ cm. For a closed cylinder, $SA=2\pi r^2+2\pi rh \approx 703.36 \text{ cm}^2$.

Likely wrong answer: Using $r=14$ and producing an answer based on 14^2 .

Misconception: Diameter is being substituted where the formula requires radius.

Next move: Write $d=14$, then $r=7$ before any formula; circle the r in the formula.

Exit Ticket Diagnostic — High-Leverage Item

Correct answer: Both contact faces become hidden, so subtract the shared face area twice.

Likely wrong answer: Subtract the contact area once.

Misconception: Only one of the two touching faces is being removed from the exposed surface count.

Next move: Shade both touching faces before adding exposed areas.

Variation Lab — Expected Ideas

1. Closed cylinder: two bases + curved surface; open-top: one base + curved surface; tube may omit both end disks depending on context.
2. Joined solids hide both contacting surfaces; subtract the contact area twice.
3. If diameter is given, halve it before using any formula written with r .

Transfer Check — Suggested Responses

Familiar: $2\pi(9)+2\pi(30)=244.92 \text{ cm}^2$. Altered: one base + curved surface = $28.26+188.4=216.66 \text{ cm}^2$. Non-routine: $2(150)-2(25)=250 \text{ cm}^2$. Repair: $r=6$; $SA=2\pi(36)+2\pi(60)=602.88 \text{ cm}^2$.

Exit Ticket — Answers

1. $SA=2\pi(2^2)+2\pi(2)(5)=87.92 \text{ cm}^2$
2. Both touching faces become internal; subtract the contact area twice.
3. $r=7 \text{ cm}$